Memory Across Eye-Movements: 1/f Dynamic in Visual Search

Deborah J. Aks,^{1,4} Gregory J. Zelinsky,² and Julien C. Sprott³

The ubiquity of apparently random behavior in visual search (e.g., Horowitz & Wolfe, 1998) has led to our proposal that the human oculomotor system has subtle deterministic properties that underlie its complex behavior. We report the results of one subject's performance in a challenging search task in which 10,215 fixations were accumulated. A number of statistical and spectral tests revealed both fractal and 1/f structure. First, scaling properties emerged in differences across eye positions and their relative dispersion (SD/M)—both decreasing over time. Fractal microstructure also emerged in an iterated function systems test and delay plot. Power spectra obtained from the Fourier analysis of fixations produced brown $(1/f^2)$ noise and the spectra of differences across eve positions showed 1/f (pink) noise. Thus, while the sequence of absolute eve positions resembles a random walk, the differences in fixations reflect a longer-term dynamic of 1/f pink noise. These results suggest that memory across eve-movements may serve to facilitate our ability to select out useful information from the environment. The 1/f patterns in relative eye positions together with models of complex systems (e.g., Bak, Tang & Wiesenfeld, 1987) suggest that our oculomotor system may produce a complex and selforganizing search pattern providing maximum coverage with minimal effort.

KEY WORDS: visual search; eye-movements; attention; Fourier analysis; pink noise; selforganized criticality.

A classic problem in the study of perception centers on our ability to perceive a stable world despite the dynamic nature of the retinal image.

¹Department of Psychology, University of Wisconsin – Whitewater, Whitewater, WI.

²Department of Psychology, State University of New York, Stony Brook, NY.

³Department of Physics, University of Wisconsin – Madison, Madison, WI.

⁴Correspondence should be directed to Deborah J. Aks, Department of Psychology, University of Wisconsin, Whitewater, WI 53190; e-mail: aksd@mail.uww.edu.

Various approaches have been taken to solve this problem, but the most common, in the eye-movement literature, appeals to at least one of three key mechanisms: First, the blurred retinal image that occurs a few times per second is thought to be masked by a mechanism of saccadic suppression. Thus, the "smearing" effect of the 30 ms saccade is eliminated through masking (e.g., Burr, 1980; Matin, 1974). A second mechanism that can account for the perceived stability of the external world is an internal process that adjusts for the shift of the retinal image by recording and compensating for the direction and extent of the saccade (e.g., MacKay, 1973; Matin, 1972; Shebilske, 1977). Finally, the third mechanism, perhaps most relevant to the present study, is that a more elaborate memory may persist across eve-movements (e.g., Irwin, 1992). Any of a number of theoretical mechanisms may explain how a record is maintained across eve movements. Examples include summation of information across visual fixations (Jonides, Irwin & Yantis, 1981), transaccadic fusion (e.g., Irwin, 1996; O'Regan & Levy-Schoen, 1983), and a map-like representation of the environment (Hayhoe, Lachter, & Feldman, 1991). Visual search models postulate similar integrative mechanisms (Palmer, 1995; Grossberg, Mingolla & Ross, 1984; Shore & Klein, in press: Wolfe, 1994).

Evidence for a record across eye-movements comes from a variety of tests including: detection of target displacement during saccades (Irwin; 1993), detection of changes across eye-contingent displays (e.g., Simons & Levin, 1997), and localization of a target defined by separately fixated, yet, superimposed display patterns (Hayhoe, Lachter & Feldman, 1991; Jonides, Irwin & Yantis, 1981). However, Horowitz & Wolfe (1998) studied memory across visual search through random repositioning of search stimuli across trials, and found no influence on reaction time (RT) from the number of items in the display (i.e., RT slopes). They concluded visual search is randomly sampled, and thus, behaves as a memoryless system.

A preponderance of additional evidence suggests that if any memory exists across eye-movements, it is extremely impoverished and by no means representative of the external world (O'Regan, 1992). At best, a modicum of spatial (i.e., Hayhoe, Lachter & Feldman, 1991) and some object information (i.e., Irwin, 1996) may be retained across fixations. It is conceivable that the only memory across eye-movements is a relatively simple one that serves to facilitate our ability to select out useful information.

EXTERNAL VS. INTERNAL INFLUENCES ON SEARCH

Empirical studies on both eye-movement and overall visual search RTs suggest that salient external information, such as high target-distractor discriminability (Duncan & Humphreys, 1989), can guide both attention and search (Motter & Belky, 1998; O'Regan, 1992). A number of researchers have gone so far as to suggest that the only memory involved in eyemovement control is in the external world (O'Regan, 1992; Gibson, 1979). Such an account is plausible when eye-movement function is regarded as "exploratory" with eye-movements serving as a probing device selecting out useful information for further use (i.e., O'Regan, 1992). Thus, "seeing" via eye-movements may simply be the action of probing the environment through changes in retinal sensations, and subsequently integrating these sensations into one's cognitive framework.

In the present study, rather than focus on external factors known to guide eye-movements, we study the internal mechanism driving search. We do so by focusing on a search condition where the external information is not sufficient to "pull" search. Nevertheless, we expect the internal mechanism to be highly sensitive to external sources, and thus highly adaptable to a variety of circumstances (Fisher, Duffy, Young & Pollatsek, 1988). We further expect such a mechanism to produce erratic, yet, deterministic search.

One compelling reason we might expect erratic search from an internally driven mechanism relates to the connection among eve movements, attention and efficient search. Attentional orienting is a reasonable candidate for driving endogenous search since it has the property of selectivity needed to achieve optimal information pick-up. Such a perspective linking eve-movements to attention is justified to the extent that the two are strongly correlated (e.g., Allport, 1987; McConkie & Rayner, 1976; Posner, 1980) as is suggested by oculomotor-readiness theories (e.g., Klein, 1980; Remington, 1980). Even though attention is not necessary to guide eye movement, as in the case of covert-orienting (Posner 1980), attention certainly has an influence (e.g., Remington, 1980; Shepard, Findlay, & Hockey, 1986). A link between attention and eye-movements is especially likely in circumstances where we voluntarily seek out a target of low-discriminability and external cues are unavailable to guide search. Thus, we test search in a challenging scene for a target that is not distinct from the surrounding background. In the real world, such a task may arise when a radiologist tries to detect a tumor in an x-ray, an air-traffic controller is on the look-out for intersecting blips (i.e., planes) on a radar screen, or a more common task may involve looking for a particular object in a cluttered environment. In all of these search tasks, the camouflaged environment provides a challenge to our visual system thus permitting us to analyze the internal mechanism driving search.

SYSTEMATIC VS. NON-SYSTEMATIC SEARCH

A variety of strategies can be used to perform efficient search in a challenging search scene. One simple yet systematic search strategy in a challenging environment may involve a binary sub-division of the visual field until the full field has been thoroughly searched (i.e., Shannon's information theory, 1948), or simply a systematic left-to-right, top-to-bottom search (e.g., Bouma & Bouhuis, 1984; Kolers, 1976). Additional strategies might include search guided by expectations (Kowler, 1989), gathering information from previous fixation (Motter & Belky, 1998; Rayner & Pollatsek, 1981), or search in a pattern resembling a random walk (Scinto, Pillamarri, & Karsh; 1986). All of these search sequences involve dependencies in eye-movements and implicate at least a short-term memory present across fixations, and, in some cases, an attentional mechanism.

Although the more systematic search strategies would successfully guide the visual system to find a target, these strategies would not be so effective if time were limited or the target location changed over time. To be executed, an "overly" systematic search requires a great deal of time and resources. A less energy-intensive search might be one that is less systematic or perhaps even random in covering the visual field. The literature documents numerous examples of what appear to be non-systematic searches (e.g., Engle, 1977; Inditsky & Bodmann, 1980; Krendel & Wodinsky, 1960). Similarly, visual search in a complex environment often is reported as erratic (i.e., Ellis & Stark; 1986; Engel, 1977; Inditsky & Bodmann, 1980; Kraiss & Knauper, 1983, or simply random (Krendel & Wodinsky, 1960; Groner & Groner, 1982; Horowitz & Wolfe, 1998).

Additional studies have demonstrated the efficiency of random search, in which successive eye fixations appear independent of one another and every location that could potentially be visited, and on each trial, all locations have an equal chance of being visited. Megaw & Richardson (1979) argue that random search provides better coverage and more efficient search than the systematic searches they observed (see also Locher & Nodine, 1974). Of four subjects, the two systematic searches took longer (but had slightly fewer misses) than two irregular search strategies. Engel (1977) and Kraiss & Knaeuper (1983) also show that systematic search for an uncertain feature target is no faster than random search. Widdel & Kaster (1981) further illustrate this in their simulation, and Scinto & Pillalamarri (1986) provide additional evidence that spontaneous human search is "nearly" random.

NON-SYSTEMATIC BUT DETERMINISTIC SEARCH: COLORED NOISE

The presence of apparently random behavior in effective visual search has led to our proposal that the human oculomotor system may have subtle self-organizing deterministic properties that can produce complex search behavior. The erratic fluctuations produced by human eye movements may be characterized by (pink) colored-noise, described below, and provide maximum coverage of the visual field at a minimum of computational cost. A pseudo-random process, governed by a simple set of rules, would be cost-effective in searching a complicated scene. Evidence for either a selforganizing complex system or a chaotic system would reflect determinism inherent to the system and support the notion that visual search does maintain memory across fixations.

A large number of natural systems including earthquakes (e.g., Bak & Tang, 1989; Paczuski & Boettcher, 1996, population dynamics (Miramontes & Rohani, 1998), and various cognitive and reaction time behaviors (Gilden, 1996; Gilden, Thornton, & Mallon, 1995) possess statistically similar dynamical properties suggestive of such complex systems. These statistical properties occur independent of the particular details of the system.

Examination of the statistical properties of these systems' fluctuations has revealed dynamics with well-defined generic scaling properties in the form of power laws (Bak Tang & Wiesnfeld, 1988). Power law relations, obtained from the Fourier transform of fluctuations, reveal long-term influences that may be the product of a simple yet flexible process: one that may be useful in searching efficiently for an item in a cluttered environment.

Behavior governed by power laws has a fractal structure with the important properties of an irregular (non-integer) shape, infinite detail, and self-similarity across all scales of the system (e.g., Mandelbrot, 1967). These systems are complex in that they consist of many interacting individual components and no single characteristic scale is best suited to describe them. In other words, there is *not* just one time scale that controls the temporal evolution of these systems: The means and variances depend on the size of the sampling resolution. Although the (dynamical) response of the systems is complex, the simplifying aspect is that the statistical properties are described by simple power laws.

The self-similarity of fractals appears in fluctuations that occur in the same proportion at all scales, and offers a high degree of statistical redundancy to permit a compact representation. The amount of information needed to be stored is reduced to a unique pattern plus a simple iterative function. Because of its compactness, fractals are currently used to store digital information (Barnsley, Devaney, Mandelbrot, Peitgen, Saupe & Voss, 1988; Watson, 1987), and also appear to be a suitable candidate for coding in the human visual system.

In a particular form of power scaling, those dominated by low frequencies, the temporal phenomenon scales as the inverse of the frequency (f), or as 1/f noise. Bak et al (1987) suggest that these systems, with a power spectral exponent of $\alpha = -1.0$ (i.e., f^{α}), consist of many interacting constituents,

are ubiquitous in nature (see Bak, 1996 and examples above), and under many conditions, are dynamical systems which organize themselves into a state with a complex but rather general structure.

One proposed model of these systems is Self-Organized Criticality (SOC⁵; Bak et al, 1987). In the SOC model, dramatic change, or *critical-ity*, occurs from the local interaction of the system's component parts. Such changes give rise to the *self-organization* within the system, wherein patterns develop in the absence of a controlling agent. The *simplicity* of the local rules with which neighbors interact is another notable feature in light of the resulting complex behavior, and the ease with which SOC can easily be generalized to a neural network that can evoke perceptual changes (Aks, Nokes, Sprott & Keane, 1998). Here local interactions can occur through lateral inhibitory and excitatory effects across neurons, and these can produce perceptual changes via threshold mechanisms (i.e., Stassinopoulos & Bak, 1995).

PREDICTION

We believe the human oculomotor system may have subtle selforganizing properties that can produce erratic fluctuations in search behavior. Furthermore, we argue that this complex behavior is a cost-effective strategy for searching a complicated scene—such a search can provide maximum coverage of the visual field at a minimum of computational cost. Evidence for such a self-organizing complex (or chaotic⁶) system would reflect determinism inherent to the system, and support the notion that visual search maintains a memory across fixations.

Recent applications of dynamical approaches to other cognitive and perceptual phenomena (Kelso, 1992; Port & Van Gelder, 1995; Pressing, 1999) show great promise for application to the visual search system. Our proposal that the human visual system may be driven by a deterministic process with subtle but important self-organizing properties is tested here in a challenging

⁵Bak's SOC theory is currently under debate as to whether it is a reliable model of 1/f dynamics. Alternative models under investigation maintain many similar properties including simple rules producing complex behaviors and self-organization (e.g., Miller, Miller & McWhorter; 1993; De Los Rios & Zhang, 1999). Thus SOC or similar alternatives could account for described data trends.

⁶Alternative to SOC, efficient search may be accomplished by a random-in-appearance, yet, chaotic process. Characteristics of chaotic determinism that make such systems a suitable candidate for a visual search mechanism are similar to those of complex systems. First, this deterministic process possesses fractal structure and has the advantage of being able to store unlimited information. Second, given the simplicity of the underlying code, a chaotic process may produce an efficient search strategy of pseudo-random sampling in a situation where the target is uncertain. A third property (unique to chaotic systems) of sensitivity to initial conditions can afford the system flexibility.

visual search task. Our analysis, described in the Methods section, focuses on the impact of time on the resulting probability distributions and power spectra. We look for scale-invariance in eye movements by evaluating whether the means and variances of these data distributions change over time, and whether their power spectra can be modeled by power laws. Finding a scale invariant perceptual system, characterized by a power law function, would suggest that there is determinism and compact coding of information in this system (i.e., Voss, 1992). Furthermore, evidence of SOC in the perceptual system, as indicated by 1/f power laws, would present another illustration of a complex system with a simple underlying dynamic—one that can potentially account for the flexibility of our visual system in adapting to novel environments.

METHOD

Subjects

One male undergraduate student from the University of Illinois participated in the visual search experiment. He received \$6.00 per hour for participating in the experiment.

Stimuli and Apparatus

The visual search task consisted of eighty-one 0.43° items. The target was an upright T, and the distractors were Ts rotated 90°, 180°, and 270° from vertical. The experiment consisted of 400 target present trials lasting approximately 2.5 hours. Search was subdivided into eight 20-min sessions with approximately 5-minute intervening rest periods. The subject's eye movements were tracked over an 18° horizontal region on a Conrac computer screen, with the overall display contained in a 754 × 497-pixel rectangle. Screen and item dimensions allowed a minimum inter-element separation needed for unambiguous fixation. Items were presented in a pseudo-random arrangement so that all locations had an equal probability of being searched. Eye movements were sampled on a Generation V dual purkinje-image (DPI) tracker that was controlled by a 486 computer.

Procedure

The subject was instructed to search for the target and to fixate it once it was found. The subject's head was stabilized with a dental bite. Each trial began with a central fixation symbol lit for 1000 ms, followed by the display, which remained visible until the observer responded. The task was to press a hand-held button when the target was located.

The duration and x and y positions of the eyes were recorded at each fixation. Each measure was treated as a set of data points whose spatial and temporal properties were analyzed over the course of search. Additional parameters of the eye-movements were used to map out the trajectory of the eyes as they moved from fixation to fixation. These included differentiation of consecutive eye-positions (e.g., $x_n - x_{n+1}$), eye movement distance $(x^2 + y^2)^{1/2}$ and eye movement direction (arctan (y/x)).

Analysis Strategy

Dynamical systems analyses are quite distinct from conventional vision analyses. While the dynamic of visual search may be captured through frequent sampling of behavior over time, the bulk of vision research examines the average across trials of the same condition, and thus provides a stationary "snap-shot" of the impact of different manipulations on behavior. In visual search studies, analyses are based on overall RTs which are a cumulative record of a sequence of processes. Search efficiency and parallel vs. serial mechanisms are thus inferred from the pattern of RTs across different set size conditions (Treisman & Gelade, 1988)—a presumption fraught with controversy (e.g., Townsend, 1971; 1976; 1990; Grossberg, Mingolla, & Ross, 1984; Humphreys, Quinlan & Riddoch, 1989; Pashler, 1987; Treisman, 1992; Wolfe, 1996). Thus, visual search assessed solely by cumulative RT analysis is necessarily an indirect assessment of underlying processes and consequently more prone to erroneous inference.

Dynamical systems approach, similar to only a handful of conventional eye movement studies, utilizes a direct numerical analysis of the data across a sequence of behaviors (i.e., a series of eye fixations). Distinguishing a random from a non-random data series is key and accomplished by looking for subtle statistical regularities (i.e., Noton & Stark, 1971; Ellis & Stark, 1986; Groner & Groner, 1984). Finding dependencies in the data can reflect general structure in a noisy system. However, as shown earlier, previous studies of eye movement behavior show mixed results, perhaps due to the diverse approaches used. Even among those studies showing nonrandom patterns, descriptions of particular patterns of visual search is limited.

To further understand potential structure underlying eye movements, we use an assortment of linear and non-linear techniques to look for patterns across data points in the series. We use analyses of probability distributions, power spectra, Lyapunov exponent, and various measures of the

fractal dimension, along with graphical display of data records with phasespace plots, return maps, and Poincaré movies. The following description of techniques is limited to linear analyses since, as described in the results, no clear trends emerge on various tests of non-linear determinism. For a review of nonlinear analyses see Hilborn (1994) or West & Deering (1995).

Our main diagnostic of temporal correlation involves performing *fast* Fourier transform (Press, Flannery, Teukolsky & Vetterling, 1986) on the fixation series and plotting the power (mean square amplitude) against frequency. A power spectrum with a few dominant frequencies shows data that can be well approximated by a Fourier series with just a few terms. Of greater interest here is whether our data can be described by a power law, and thus possesses scale invariance associated with a complex system. A linear function on a double-log plot indicates the presence of a power law. The regression slope of this function determines the power exponent. When the exponent of the spectrum power law is $\alpha = -1.0$, the given temporal phenomenon scales as the inverse of the frequency (f) or as "1/f noise." In this particular form of scaling, fluctuations occur in the same proportion at all scales (ie., they are self-similar and scale invariant), and there exists a great deal of fine structure in the data.

An important aspect of the spectral analysis is that it provides a useful measure of the strength of memory across the system. This is important not only in assessing whether memory exists across eye-movements but it quantifies its strength in terms of the exponent making up the power law. The steepness of the slope (on a log-log scale) reflects the duration of memory (i.e., correlation across points): Brown $1/f^2$ noise has a steep slope indicating short-term correlation. Pink $1/f^1$ noise has a shallow slope indicating extremely long time correlation, and white $1/f^0$ noise with a flat spectrum indicates no correlation across data points.

The power spectra provide similar information obtained through *auto-correlation* procedures. Consider a series of time signals. The signal fluctuates up and down in a seemingly erratic way. Does the signal at t_0 influence what is measured at a later time $t_0 + t$? We are not interested in any specific time instant t_0 but rather in the typical (i.e., the statistical) properties of the fluctuating signal. The amount of dependence, or history in the signal can be characterized by the temporal correlation function. If there is no statistical correlation G(t) = 0. The speed with which G(t) decreases measures the duration of the correlation or memory in the signal.

Iterated Function Systems (IFS) also provide sensitive tests for deviations from randomness (Peak & Frame, 1994; Jeffrey, 1992; Sprott & Rowlands, 1995; Mata-Toledo & Willis, 1997). The technique involves first sorting the data from the minimum to the maximum value and then subdividing the series into four segments in such a way that each segment contains the same number of points. The original unsorted data set is then normalized and course-grained into four values, 1 to 4 representing the quartile to where the data belong. The representation space is a square where the four corners are labeled 1 to 4 in a clockwise direction (starting in the lower left corner). Each value of the coarse-grained series is associated with the corner having the same number. A point is plotted halfway between the center of the square and the first point of the series. A second point is plotted half way between the first plotted point and the second point in the series and so forth.

The graphical output of the IFS procedure is clumped patterns when data contain colored noise and homogeneously filled spaces when the data are uncorrelated. White $1/f^0$, pink $1/f^1$ and brown $1/f^2$ noise, are easily distinguished. White noise is a space-filled uncorrelated process that uniformly fills its space of representation. At the other extreme, Brown noise accumulates over the diagonals and some of the sides of the square leaving most of the representation space empty. 1/f noise produces self-similar repeating triangular structures of different sizes and accumulates, albeit in a dispersed way, near the diagonals. Through visualization of the fine structure of the time series, the IFS test can reveal correlation in the data, thus assisting in characterizing the color of noise in a system.

RESULTS

Visual search produced, on average, 24 fixations (SD = 15) per trial with each trial lasting 7.6 seconds (SD = 6.9 sec). Mean fixation duration was 212 ms (SD = 89 ms) with 10,215 fixations across the complete search experiment. The number of fixations decreased from 1888 to 657 across the eight sessions with fixation duration tending to increase from 206 to 217 ms. Mean deviation from last trial fixation to new target location was 0.4° visual angle indicating a high degree of accuracy in actual target detection.

Figure 1 shows a scatter plot of eye fixations corresponding to horizontal (x) and vertical (y) screen coordinates. There are clear clusters of fixations in the center and near the boundaries, and intermittent gaps throughout. Figure 2 shows a representative sample of the first differences across eye position $(y_{n+1} - 1/n)$. Trends were similar for x and y eye coordinate positions. While differences across y eye positions gradually increased over time, differences across x eye positions tended to decrease over time. The same trends occurred with relative dispersions (SD/M; Liebovitch, 1998)—a measure which reflects system contingencies as a function of sampling resolution. Such changes in means and variance with fixation duration are characteristic of fractal structures.



Fig. 1. Scatter plot of 10,215 eye fixations for the entire visual search experiment. Eye fixations are represented across horizontal (x) and vertical (y) screen coordinates in pixel units. There are clear clusters of fixations in the center and near the boundaries, and intermittent gaps throughout the display.

The delay plot in Fig. 3 shows a diffuse but positively correlated structure among adjacent data points that resembles colored noise (r = .90). Superimposed on this pattern, is a clustering of adjacent fixations in the central region and along the horizontal and vertical axes. The concentration of data along the *x*-axis is the result of fixating the center of the screen at the start of each trial with subsequent fixations falling along the wider band. Removing the first three fixations for each trial eliminated this artifact but did not affect any of the remaining analyses.

Fourier analysis of x and y fixations produced brown $(1/f^2)$ noise as shown in Fig. 4. The brown noise persisted with or without the initial central fixations, and was present in each of the eight individual sessions, although there was some flattening of regression slopes at the extremes of the spectra. Figure 5, shows the results of the IFS Clumpiness test. Clustering along the diagonals reveals short-term, highly correlated consecutive data points typically found in brown noise. The additional fractal microstructure appearing



Fig. 2. A representative sample of the fixation series for the first differences of eye position $(y_{n+1} - y_n)$. Only fixations along the vertical coordinate are shown. The erratic pattern in the fixation series are similar for horizontal eye positions.

in the IFS test reflects long-term but weaker correlation often associated with pink noise.

The mean regression slope of the power spectra for the x and y eyeposition coordinates was $\alpha = -.23$. This relatively shallow slope is due to the spurious low frequency region. Differentiating data increases the steepness of the regression slopes to approximately $\alpha = -.7$ with the steepest slopes persisting in the high frequency region of the curve. These 1/f trends are illustrated in the power spectra of Fig. 6, and the IFS Clumpiness test shown in Fig. 7. A combined measure of distance across eye fixations $(\Delta x^2 + \Delta y^2)^{1/2}$ produced power spectra with 1/f trends dominating the lower frequency range, and $1/f^2$ trends dominating the high frequency range (Mean $\alpha =$ -.47; Fig. 8). The corresponding IFS test, shown in Fig. 9, produced a clear but distinct colored noise pattern with more diffuse clustering of data points than those found in the raw and differentiated data sets. Random shuffling



Fig. 3. The delay plot of highly correlated consecutive eye-fixations (r = .9). Superimposed on this pattern, is a clustering of data in the central region and along the horizontal and vertical axes. Data points are bounded by dimensions corresponding to those of the search screen. Concentration of data along the horizontal-axis are the result of fixating the center of the screen at the start of each trial with subsequent fixations falling along the wider band.

of x, y and distance data sets produced white noise. Thus, while the sequence of absolute eye positions resembles a random walk, the differences in these fixations possess a potentially important, longer-term dynamic characterized by 1/f pink noise.

To assess whether any of these trends were due to a deterministic search versus a random search constrained by the boundary of the screen, we simulated random eye movements on the computer with the simple constraint to reverse the direction of movement if it exceeded the boundaries of the display. The computer-generated data produced the same probability distribution as the human data and clear $1/f^2$ trends. However, when differentiated, the simulated data produced flat spectra characteristic of white noise, whereas human data produced trends resembling pink-noise. Thus, it



Fig. 4. Power spectra of vertical eye fixation series from the entire visual search experiment. Total power equals mean squared amplitude. Brown $(1/f^2)$ noise trends emerge. Also shown is a line depicting an exact $1/f^2$ power spectrum.

appears that our search system does not simply reduce to a random walk constrained by boundary conditions.

Three additional parameters of the eye-movements examined were direction and duration of the eye fixations, and duration of saccades between fixations. Power spectra on all three produced relatively flat $(1/f^0)$ power spectra. Differentiated data produced f/1 trends for direction and duration of fixation and white $(1/f^0)$ noise in the saccades. Finally, no clear trends emerged on various tests of nonlinear determinism. The lower limit estimate of the fractal dimension was D = 4.0 suggesting that visual search for an item of unique orientation entails a highly complex system. If nonlinear dynamics do exist in the search system under study, they are difficult to decipher.

IFS Clumpiness Test



Fig. 5. An IFS Clumpiness test is applied to the x and y fixation series with the latter shown here. This technique is used to create a pattern that helps to visually characterize the color of the noise since it produces clumped patterns for colored noise while producing homogeneously filled spaces when the data is uncorrelated. In the IFS test, we start at an initial point, and then read in the data, plotting a succession of points according to the iterative rule (described in the text). The result is a scattering of points in the plane. The plot represents a trajectory, since the position of each point is determined by all the previous points. Each point gives a short-term history as the influence of previous fixations diminish over time. Any departure from a uniform distribution of points is evidence for determinism. Clustering along the diagonals in the figure, reveals the short-term highly correlated pattern associated with brown noise. The additional fractal microstructure reflects longer-term, but weaker correlations often associated with pink noise.

DISCUSSION

The dynamical systems approach offers novel theoretical ideas and tools to help us understand a variety of psychological phenomena (Gilden, 1996; Gilden, Thornton, & Mallon, 1995; Pressing, 1999). Here, we examined eye



Fig. 6. Power spectra of first differences of x and y fixation series produced pink (1/f) noise. Regression slopes of the power spectra are $\alpha = -.6$ in the high frequency region of the curve. Overall slopes were $\alpha = -.23$ including the spurious low frequency region. Also shown is a line depicting an exact 1/f power spectrum.

movement behavior using this approach to explore the underlying mechanisms guiding complex visual search. We predicted that the human visual system might produce a complex, yet deterministic, search when explicit information is unavailable to guide the eyes to a target. We use dynamical systems analyses to present a novel perspective to the study of eye movement behavior, and to contribute the new insights to the debate in the visual-cognitive literature which is currently focused on whether memory plays a role in guiding visual search (e.g., Horowitz & Wolfe, 1998; Kristjansson, 2000; Shore & Klein, in press).

Memory-based theories of scanning suggest, as common sense might, that when searching for an item, the rejected items should be noted in some fashion so that effort is not expended in re-examining already searched items.

IFS Clumpiness Test



Fig. 7. Results of the IFS Clumpiness test of differentiated vertical (y) fixations. A similar pattern emerged for horizontal (x) fixations. The diffuse fractal microstructure reflects longer-term correlations appearing weaker than those in the raw data. Both cases resemble patterns associated with pink noise.

Such a mechanism is often referred to as "inhibitory tagging" (e.g., Klein, 1988; Klein & MacInnes, 1999). Moreover, popular theories of visual perception suggest that information about the identity of objects is accumulated over time (Treisman & Gelade, 1980; Grossberg, Mingolla & Ross, 1994). According to both sets of theories, search utilizes information from previous fixations to guide subsequent search.

Horowitz & Wolfe (1998) recently challenged the assumption that search proceeds through either inhibitory tagging or identification of previously searched items (as summarized in the introduction). Their findings of RTs being unaffected by randomly repositioned items, together with recent research showing that visual memory is often surprisingly poor (Rensink,



Fig. 8. Power spectra of distance across eye fixations $(\Delta x^2 + \Delta y^2)^{1/2}$. Pink 1/f trends are dominant in the lower frequency range and $1/f^2$ trends are dominant in the high frequency range (Mean $\alpha = -.47$). Also shown is a line depicting an exact 1/f power spectrum.

O'Regan, & Clark, 1997; Simons & Levins, 1997), led to their proposal that the visual system retains little information about the locations (or identity) of objects over time, and instead acts on fleeting neural representations that are overwritten by a change in the visual scene (Horowitz & Wolfe; 1998).

The argument that visual search does not keep track of the previously searched spatial locations comes from examination of overall RTs of visual search. Reliance on such a coarse measure of behavior means that subtle contingencies in scanning behavior may be overlooked. These less obvious forms of memory may be revealed instead through a direct analyses of the eye movements themselves. Here, we ask whether memory exists in visual

IFS Clumpiness Test



Fig. 9. Results of the IFS Clumpiness test of distance between fixations $(\Delta x^2 + \Delta y^2)^{1/2}$. A unique colored noise pattern emerges with more diffuse clustering of data points than those found in the raw data set.

search using a direct and intensive analysis of the eye movement data from a single subject.⁷

⁷Our in-depth focus on a single data set is also relevant to the question of generalizability, and the possibility that our male undergraduate subject might possess eye movements unique from individuals representing other sectors of the population. However, an intensive focus on such a single data set, and similar such sets in the field of nonlinear dynamics, presents a unique situation in that the dynamical systems approach applies analysis tools that are only relevant to individual analysis. Combining data sets, as is typically done in conventional analyses, can be problematic for dynamical analyses in that the relevant unit of analysis, variability, is collapsed across the group of subjects. The result is a loss of information that may be critical to detecting complex data patterns. Only after a baseline performance is established from intensive focus on a single-subject, should research be undertaken to explore the eye movements across different individuals. Such future research will serve to evaluate the robustness of the present findings and potentially uncover some interesting individual differences. Perhaps children well-versed in video arcades or radiologists accustomed to challenging searches for tumors in ct-scans might show search patterns distinct from individuals rarely exposed to such tasks.

Our key finding that a sequence of fixations can be represented by colored noise and a power law function, confirmed our prediction that search may be guided by memory. Contrary to Horowitz & Wolfe (1998), we found search behavior is *not* random and that contingencies do exist across fixations. While much cognitive theory implicates tagging and inhibition of return to previously visited items, the evidence of memory that we have found involves general contingencies across fixations separate from any influence on overall search speed. Furthermore, the power law functions—1/f and $1/f^2$ —found in search, show the system has scale invariant properties typically associated with a system optimized to adapt to a changing environment. Since systems characterized by power functions are known to be flexible, this suggests that the contingencies guiding search may play an important role in selection of appropriate information in a dynamic array of constantly changing environmental information.

Differences found in α in the $1/f^{\alpha}$ trends across the relative vs. absolute fixations ($\alpha = 1$ vs. 2) further suggest differences in underlying mechanisms (e.g., Musha, Sato & Yamamoto, 1991) as well as differences in duration of the memory across search. The brown $1/f^2$ noise, or random walk, that dominated the sequence of raw fixations likely results from constraints of the physical movements of the subjects' eyes—the sequential nature of our eyemovements forces adjacent fixations to be relatively close to one another. Additional support for such a random walk in raw eye fixations were found in Scinto, Pillalamarri & Karsh's (1986) eye movement model, and in our simulation of random eye movements having only the single constraint of reversing direction when a random movement exceeds the boundaries of the search display.

Neither model, however, produces the 1/f behavior that emerged in our results (Figs. 6 to 9). In these data, power spectra and IFS tests on the distance and direction across fixations revealed pink 1/f noise. These results suggest a long-term memory is maintained across the complicated search in a manner that may involve use of a simple set of rules with self-organizing properties (i.e., Bak Tang & Wiesenfeld, 1988).

The manner in which these rules operate can be understood using Bak's SOC sandpile model, as well as a neural network model where the constituents (i.e., grains of sand or neuronal activation) can be represented as a two-dimensional grid of interacting cells. In the case of a neuronal SOC model, each cell possesses a certain degree of activation perhaps induced by movement of the eyes to different locations. Spatially adjacent activation, on the corresponding neural network, can be represented by a numerical value, Z(x, y). As individual neurons are activated beyond a threshold say of 4, the activity in the original site is dispersed to surrounding cells, incrementing the activity in these regions by 1, $\{Z(x, y) \rightarrow Z(x, y) + 1\}$, and thus depleting

the activity in the original site to zero, $Z(x, y) \rightarrow Z(x, y) - 4$. In the absence of useful environmental information during visual search, the eyes may be guided to sites that contain the highest level of activity among immediately surrounding cells, and evade local sites depleted in neuronal activity. The global result can be a complicated search pattern that could easily be mistook for a random search. Thus, finding 1/f noise in eye movements illustrates that search is not random, and instead may result from guidance of eye movements by changes in intensity of neuronal activity across the network of neurons. Such a model implicates a simple form of spatial memory existing across the sequence of eye movements.

The 1/f eye movements may also involve a cognitive mechanism such as attention-based sampling and selection of useful information from a complicated environment. Whether neuronal interaction involving the spread of activation drives this selection process is an open question as is how such a system can produce the rapid and effective search known to occur in humans. The answer may relate to the general finding that 1/f systems offer an optimal compromise between efficient recovery of information and the tendency to err (Voss, 1992). The significance of these complex yet adaptive behaviors remains open to future scientific inquiry.

REFERENCES

- Aks, D. J., Nokes, T. Sprott, J. C. & Keane, E. (1998). Resolving perceptual ambiguity in the Necker Cube: A dynamical systems approach. *Abstracts of the Psychonomics Society*, 3, 38.
- Allport, D. A. (1987). Selection-for-action: Some behavioral and neurophysiological considerations of attention and action. In H. Heuer & A. F. Sanders, (Eds.), *Perspectives on Perception and Action*. Hillsdale, N.J.: Erlbaum.
- Bak, P. (1996). *How nature works: the science of self-organized criticality.* (pp. 52–64) New York: Springer-Verlag.
- Bak, P., & Tang, C. (1989). Earthquakes as a self-organized critical phenomenon. Journal of Geophysics Research—Solar. Earth Planet, 94, 15635–15637.
- Bak, P., Tang, C., & Wiesenfeld, K. (1987). Self-organized criticality: An explanation of 1/f noise. Physical Review Letters, 59, 381–384.
- Bak, P., Tang, C., & Wiesenfeld, K. (1988). Self-organized criticality. *Physical Review A*, 38, 364–374.
- Barnsley, M. F., Devaney, B. B., Mandelbrot, H. O., Peitgen, D., Saupe, D., & Voss, R. F. (1988). *The Science of Fractal Images.* New York: Springer-Verlag.
- Bouma, H., & Bouhuis, D. G. (1984). Components of visual attention. Attention and Performance X. Hillsdale, NJ: Erlbaum.
- Bovik, A. C., Clark, M. & Geisler, W. (1990). Multichannel texture analysis using localized spatial filters, *IEEE Pattern Analysis and Machine Intelligence*, 12, 55–73.
- Burr, D. (1980). Motion smear. Nature, 284, 164-165.
- Carlson-Radvansky, L. & Irwin D. (1995). Memory for structural information across eye movements. Journal of Experimental Psychology: Learning, Memory & Cognition, 21, 1441–1458.
- Daugman, J. G. (1991). Self-similar oriented wavelet pyramids: Conjectures about neural non-orthogonality. In A. Gorea (Ed.), *Representations of Vision*. (pp. 27–46) Cambridge: Cambridge University Press.

- Duncan, J., & Humphreys, G. W. (1989). Visual search and stimulus similarity. Pscyhology Review, 96, 433–458.
- Ellis, S. R., & Stark, L. (1986). Statistical dependency in visual scanning. *Human Factors*, 28, 421–438.
- Engel, F. L. (1977). Visual conspicuity, visual search, and fixation tendencies of the eye. Vision Research, 17, 95–108.
- Fisher, D. L., Duffy, S., Young, C., & Pollatsek, A. (1988). Understanding the central processing limit in consistent-mapping visual search tasks. *Journal of Experimental Psychology: Human Perception and Performance*, 14, 253–266.
- Gibson, J. J. (1979). The ecological approach to visual perception. Boston: Houghton Mifflin.
- Gilden, D. L. (1996). Fluctuations in the time required for elementary decisions. *Psychological Science*, 8, 296–302.
- Gilden, D. L., Thornton, T., & Mallon, M. (1995). 1/f noise in human cognition. Science, 267, 1837–1839.
- Gould, J. D. (1973). Eye movements during visual search and memory search. Journal of Experimental Psychology, 98, 184–195.
- Grieger, B. (1992). Quaternary climatic fluctuations as a consequence of self-organized criticality. *Physica A*, 191, 51–56.
- Groner, R., & Groner, R., (1982). Towards a hypothetico-deductive theory of cognitive activity. In R. Groner & P. Fraisse (Eds.) Cognition and Eye Movements. (pp. 100–121) Amsterdam: North Holland Publishing.
- Groner, R., & Groner, M. (1984). A stochastic hypothesis testing model for multi-term series problems, based on eye fixations. In R. Groner, C. Menz, D. F. Fisher, & R. A. Monty (Eds.), *Eye Movements and Psychological Functions: International Views.* (pp. 257–274). Hillsdale, NJ: Erlbaum.
- Grossberg, S., Mingolla, E., & Ross, W. D. (1984). A neural theory of attentive visual search: Interactions of boundary, surface, spatial, and object representations. *Psychological Review*, 101, 470–489.
- Hayhoe, M., Lachter, J., & Feldman, J. (1991). Integration of form across saccadic eye movements. *Perception*, 20, 393–402.
- Hilborn, R. C. (1994). Chaos and Nonlinear Dynamics: An introduction for Scientists and Engineers. Oxford: Oxford University Press.
- Hochberg, J. (1968). In the minds eye. In R. N. Haber (Ed.), Contemporary theory and research in visual perception. (pp. 309–331). New York: Holt, Reinhart & Winston.
- Horowitz, T. S. & Wolfe, J. M. (1998). Visual Search has no memory. Nature, 357, 575–577.
- Humphreys, G. W., Quinlan, P. T., & Riddoch, M. J. (1989). Grouping processes in visual search: Effects with single and combined-feature targets. *Journal of Experimental Psychology: General*, 118, 258–279.
- Inditsky, B., & Bodmann, H. W. (1980). Quantitative models of visual search. In Proceedings of the 19th symposium of CIE. (pp. 197–201). Paris: Commission Internationale de l'Eclairage.
- Irwin, D. E. (1992). Memory for position and identity across eye movements. Journal of Experimental Psychology: Learning, Memory, and Cognition, 18, 307–317.
- Irwin, D. E. (1993). Memory for spatial position across saccadic eye movements. In G. d'Ydewalle & J.V. Van Rensbergen (Eds.), *Perception and Cognition*. (pp. 323–332)
- Irwin D. E. (1996). Integration and accumulation of information across saccadic eye movements. In Inui, T., & McLelland, J. L. (Eds.) Attention and performance XVI: Information integration in perception and communication(pp. 125–155). London: MIT Press.
- Jeffrey, H. J. (1992). Chaos game visualization of sequences. Computers & Graphics, 16, 25-33.
- Jonides, J., Irwin, D. E., & Yantis, S. (1981). Integrating visual information from successive fixations. Science, 215, 192–194.
- Kelso, S. (1992). Dynamic Patterns. Cambridge: MIT Press.
- Klein R. (1988). Inhibitory tagging system facilitates visual search. Nature, 334, 430-431.
- Klein R. (1980). Does oculomotor readiness mediate cognitive control of visual attention? In R. S. Nickerson (Ed.), Attention and Performance VIII, (pp. 259–276). Hillsdale, NJ: Erlbaum.

- Klein R. & MacInnes, W. J. (1999). Inhibition of return is a foraging facilitator in visual search. Psychological Science, 10, 346–352.
- Kolers, P. A. (1976). Buswell discoveries. In R. A. Monty & J. W. Sanders (Eds.), Eye Movements and Psychological Processes. (pp. 373–395). Hillsdale, N.J.: Erlbaum.
- Kowler, E. (1989). Cognitive expectations, not habits, control anticipatory smooth oculomotor pursuit. Vision Research, 29, 1049–1057.
- Kraiss, K. F., & Knaeuper, A. (1983). Using visual lobe area to predict visual search time. *Human Factors*, 24, 673–682.
- Krendel, E. S., & Wodinsky, J. (1960). Search in an unstructured visual field. Journal of the Optical Society of America, 50, 562–568.
- Kristjansson, A. (2000). In search of Remembrance: Evidence for memory in visual search. Psychological Science, 11, 328–332.
- Liebovitch, L. S. (1998). Fractals and chaos: Simplified for the life sciences. Oxford: Oxford University Press.
- Locher, P. J., & Nodine, C. E. (1974). The role of scanpaths in the recognition of random shapes. Perception & Psychophysics, 15, 308–314.
- MacKay, D. M. (1973). Visual stability and voluntary eye movements. In R. Jung (Ed.), Handbook of sensory and voluntary eve movements. Vol. VII/3A (pp. 307–331). Berlin: Springer.
- Mandelbrot, B. B. (1967). How long is the coast of Britain? Statistical self-similarity and Fractal Dimension. *Science*, *156*, 636–638.
- Mandelbrot, B. B. (1982). The fractal geometry of nature. San Francisco: Freeman.
- Mata-Toledo, R. A., & Willis, M. A. (1997). Visualization of random sequences using the Chaos Game algorithm. *The Journal of Systems and Software*, *39*, 3.
- Matin, L. (1972). Eye movements and perceived visual direction. In D. Jameson & L.V. Hurvich (Eds.), *Handbook of Sensory Physiology*, Vol VII/4 Visual Psychophysics (pp. 331–380). Berlin: Springer-Verlag.
- Matin, L. (1974). Saccadic supression: A review and analysis. Psychological Bulletin, 81, 899– 917.
- Maylor, E. (1985). Facilitatory and inhibitory components of orienting in visual space. In M. I. Posner & B. B. Marin (Eds.), *Attention and performance XI* (pp. 189–204). Hillsdale NJ: Erlbaum.
- Maylor, E. A., & Hockey, R. (1985). Inhibitory component of externally controlled covert orienting in visual space. *Journal of Experimental Psychology: Human Perception & Performance*, 11, 777–787.
- McConkie, G. W. & Rayner, K. (1976). Identifying the span of the effective stimulus in reading: Literature review and theories of reading. In H. Singer & R.B. Ruddell (Eds.), *Theoretical models and processes of reading*. (pp. 137–162) Newark, DE: International Reading Association.
- Megaw. E. D. & Richardson, J. (1979). Target uncertainty and visual scanning strategies. *Human Factors*, 21, 303–315.
- Miller, S. L., Miller, W. M., & McWhorter, P. J. (1993). Extremal dynamics: A unifying physical explanation of fractals, 1/f noise and activated processes. *Journal of Applied Physics*, 73, 2617–2628.
- Miramontes, O., & Rohani, P. (1998). Intrinsically generated coloured noise in laboratory populations. Proceedings of Royal Society of London B. 265, 785–792.
- Monk, T. H. (1976). Target uncertainty in applied visual search. Human Factors, 18, 607-612.
- Motter, B. C. & Belky, E. J. (1998) Guidance of eye movements during active visual search. Vision Resarch 38, 1805–1815.
- Musha, T., Sato, S. & Yamamoto, M. (1991). Noise in physical systems and 1/ffluctuations. Tokyo: Ohmsha Ltd.
- Neisser, U. (1967). Cognitive psychology. New York: Appleton.
- Noton, D. & Stark, L. (1971). Scanpaths in saccadic eye movements while viewing and recognizing patterns. Vision Research, 11, 929–942.
- O'Regan, J. K. (1992). Solving the "real" mysteries of visual perception: The world as an outside memory. *Canadian Journal of Psychology*, 46, 461–488.

- O'Regan, J. K., & Levy-Schoen, A. (1983). Integrating visual information from successive fixations: Does trans-saccadic fusion exist? *Vision Research*, 23, 765–769.
- Paczuski, M., & Boettcher, S. (1996). Universality in sandpiles, interface depinning, and earthquake models. *Physical Review Letters*, 77, 111.
- Palmer, J. (1995). Attention in visual search: Distinguishing four cases of a set-size effect. Current directions in Psychological Science, 4, 118–123.
- Pashler, H. (1987). Detecting conjunctions of color and form: Reassessing the serial search hypothesis. *Perception and Psychophysics*, 41, 191–201.
- Peak, D., & Frame, M. (1994). Chaos Under Control: The Art and Science of Complexity. New York: W. H. Freeman.
- Peitgen, H. O., Jurgens, H., & Saupe, D. (1993). Fractals for the Classroom. New York: Springer-Verlag.
- Port, R. F. & van Gelder, T. (1995). *Mind as Motion: Explorations in the dynamics of cognition.* Cambridge: MIT Press.
- Posner, M. (1980). Orienting of attention. Quarterly Journal of Experimental Psychology, 32, 3–25.
- Posner M. & Cohen, Y. (1984). Components of visual attention. In H. Bouma & D.G. Bouhuis (Eds.), Attention and Performance X. Hillsdale NJ: Erlbaum.
- Pratt, J. (1995). Inhibition of return in a discrimination task. *Psychonomic Bulletin and Review*, 2, 117–120.
- Press, W. H., Flannery, S. A., Teukolsky, S. A., & Vetterling, W. T. (1986). Numerical Recipes. New York: Cambridge University Press.
- Pressing, J. (1999) The referential dynamics of Cognition and action. *Psychological Review*, *106*, 714–747.
- Rayner, K. & Pollatsek, A. (1981). Eye movement control during reading: Evidence for direct control. *Quarterly Journal of Experimental Psychology: Human Experimental Psychology*, 33A, 351–373.
- Remington, R. W. (1980). Attention and saccadic eye movements. Journal of Experimental Psychology: Human Perception and Performance, 6, 726–744.
- Rensink, R., O'Regan, J. K. & Clark, J. J. (1997). To see or not to see: the need for attention to perceive changes in scenes. *Psychological Science*, 8, 368–373.
- Rhodes, C. J., & Anderson, R. M. (1997). Epidemic thresholds and vaccination in a lattice model of disease spread. *Theory of Population Biology*, 52, 101–106.
- Rizzolatti, G., Riggio, L., Dascola, I., & Umilta, C. (1987). Reorienting attention across the vertical and horizontal meridians: Evidence in favor of a premotor theory of attention. *Neuropsychologia*. 25, 31–40.
- Scinto, L., Pillalamarri, R., & Karsh, R. (1986). Cognitive strategies for visual search. Acta Psychologica, 62, 263–292.
- Shannon, C. E., & Weaver, W. (1949). The mathematical theory of communication. Urbana: University of Illinois Press.
- Shebilske, W. (1977). Visuomotor coordination in visual direction and position constancies. In W. Epstein (Ed.) Stability and constancy in visual perception: Mechanisms and processes (pp. 23–70). New York: John Wiley.
- Shepard, M., Findlay, J. M., & Hockey, R. J. (1986). The relationship between eye movements and spatial attention. *Quarterly Journal of Experimental Psychology*, 38A, 475– 491.
- Shore, D. I. & Klein, R. M. (in press). On the manifestations of memory in visual search, Spatial Vision.
- Simons, D. J. & Levins D. (1997). Change blindness. Trends in Cognitive Science, 1, 261–267.
- Sprott, J. C., & Rowlands, G. (1995). Chaos Data Analyzer. Raleigh, NC: Physics Academic Sofware (American Institute of Physics).
- Stassinopoulos, D., & Bak, P. (1995). Democratic Reinforcement. A principle for Brain function. Physical Review E, 51, 5033.
- Tipper, S. P., Driver, J., & Weaver, B. (1991). Object-centered inhibition of return of visual attention. Quarterly Journal of Experimental Psychology, 43A, 289–298.

- Townsend, J. T. (1971). A note on the identification of parallel and serial processes. Perception and Psychophysics, 10, 161–163.
- Townsend, J. T. (1976). Serial and within-stage independent parallel model equivalence on the minimum completion time. *Journal of Mathematical Psychology*, *14*, 219–239.
- Townsend, J. T. (1990). Serial and parallel processing: Sometimes they look like Tweedledum and Tweedledee but they can (and should) be distinguished. *Psychological Science*, *1*, 46–54.
- Treisman, A. (1992). Spreading suppression or feature integration? A reply to Duncan and Humphreys (1992). Journal of Experimental Psychology: Human Perception and Performance, 18, 589–593.
- Treisman, A., & Gelade, G. (1988). A feature integration theory of attention. Cognitive Psychology, 12, 97–136.
- Umilta, C. & Moskovitch, M. (1994). Space and selective attention. Attention and Performance XV (pp. 231–265). Cambridge, MA: MIT Press.
- Voss, R. F. (1992). Evolution of long-range fractal correlations and 1/f noise in DNA base sequences. *Physics Review Letters*, 68, 3805–3808.
- Watson, A. B. (1987). Efficiency of an image code based on human vision. Journal of the Optical Society of America, A4, 2401–2417.
- West, B. J. & Deering, B. (1995). The lure of modern science: Fractal thinking. Studies of Nonlinear Phenomena in Life Sciences. Singapore: World Scientific.
- Widdel, H. & Kaster, J. (1981). Eye movement measurement in assessment and training of visual performance. In J. Moraal & K.F. Kraiss (Eds.), *Manned systems design: Methods,* equipment, and applications. (pp. 251–270). New York: Plenum Press.
- Wolfe, J. M. (1994). Guided search 2.0: A revised model of visual search. Psychonomic Bulletin & Review 1, 202–238.
- Wolfe, J. M. (1998). Visual search. In H. Pashler (Ed.), Attention, (pp. 13–71). London: University College London Press.